MA 161: Lesson 30

Antiderivatives (4.9)

Undoing the derivatives

Indefinite integral

Evaluating indefinite integrals

Coming up Next: Approximating area under curves (5.1)

Announcements

Exam 3 on Nov 20th - Lesson 18 (3.10) to Lesson 30 (4.9)

Study guide and instructions for exam 3 posted on brightspace

Office Hours

M, W, F: 245pm - 415pm

Warmup: find functions $F(x)$ and $G(x)$ such that
$F'(x) = 2x$, $G'(x) = \sin x$

$$F \xrightarrow{\text{derivatives}} 2x \xrightarrow{\text{derivative}} 2$$

x^2

$x^2 + 7$

$x^2 - 85$

$x^2 + 987659$

$$G \xrightarrow{\text{derivative}} \sin x \xrightarrow{\text{derivative}} \cos x$$

$-\cos x$

$-\cos x + 97$

$-\cos x - 873$

$-\cos x + 17958$

Antiderivative

$F(x)$ such that $F'(x) = f(x)$ is called antiderivative of $f(x)$

$$\left.\begin{array}{l} x^2 \\ x^2 - 85 \\ x^2 + 7 \\ x^2 + 98758 \\ x^2 + \underbrace{C}_{\text{Any Number}} \end{array}\right\} \text{Antiderivatives of } 2x$$

$$\left.\begin{array}{l} -\cos x \\ -\cos x + 97 \\ -\cos x - 857 \\ -\cos x + C \end{array}\right\} \text{Antiderivatives of } \sin x$$

$F(x)$ & $G(x)$ are two antiderivatives of $f(x)$, they differ only by a constant

$$F(x) = G(x) + C.$$

Indefinite Integral

A notation for antiderivatives

$$\int f(x)\, dx = F(x) + C$$

elongated S means "sum" - Later

integrand "the function whose antiderivative we are trying to find"

$F(x)$ is one antiderivative of $f(x)$

C: Constant of integration

x is the variable of integration

eg:-

$$\int 2x \, dx = x^2 + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int x^n \, dx$$

idea! Derivative Reduces the power by 1

$$\frac{d}{dx} x^4 = 4x^3 \rightsquigarrow \underbrace{\frac{d}{dx}\left(\frac{x^4}{4}\right) = x^3}$$

$\frac{x^4}{4}$ is Antiderivative of x^3

$$\int x^3 \, dx = \frac{x^4}{4} + C$$

$$\frac{d}{dx}\left(\frac{x^8}{8}\right) = x^7 \rightsquigarrow \int x^7 \, dx = \frac{x^8}{8} + C$$

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1}$$

$n = -1$

$$\boxed{\int \frac{1}{x} \, dx = \ln|x| + C}$$

Antiderivatives of some common functions

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

Derivative of Right hand side is the integrand

* find $\int 5x^3 + 7x^{-2}\,dx$

Antiderivative

$\frac{x^{3+1}}{3+1} = \frac{x^4}{4}$

Antiderivative

$\frac{d}{dx}\left(x^{-1}\right) = -1x^{-2}$

$-x^{-1}$ is Antiderivative of x^{-2}

we know $\left(k_1 f(x) + k_2 g(x)\right)' = k_1 f'(x) + k_2 g'(x)$

$$\frac{d}{dx}\left(5\frac{x^4}{4} - 7x^{-1}\right) = 5x^3 + 7x^{-2}$$

$$\int 5x^3 + 7x^{-2}\,dx = 5\frac{x^4}{4} - 7x^{-1} + C.$$

* find $\int 9\sqrt{x} - 8\sec^2 x + \frac{1}{5} e^x \, dx$

Antiderivate

e^x

Antiderivate

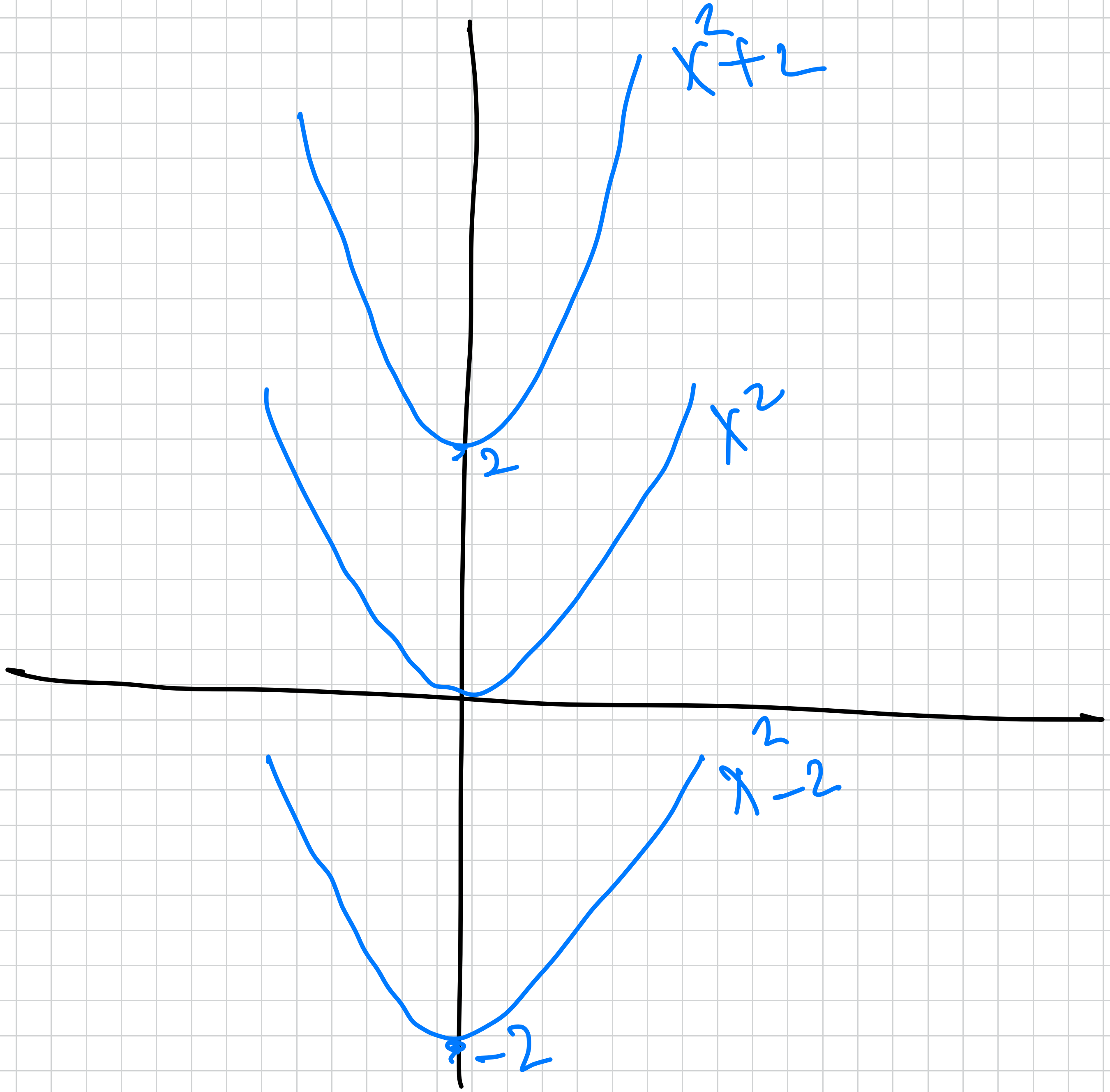
$\tan x$

Antiderivate

$\frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$

$$\int 9\sqrt{x} - 8\sec^2 x + \frac{1}{5} e^x \, dx = 9 \cdot \frac{2}{3} x^{3/2} - 8\tan x + \frac{1}{5} e^x + C.$$

find $F(x)$ such that $\underbrace{F'(x) = 2x}_{F(x) = x^2 + C}$ infinitely many possible function



If we impose an additional condition

$F(0) = 7$

$F(x) = x^2 + C \rightsquigarrow \underbrace{7 = F(0) = 0 + C}_{C = 7}$

$F(x) = x^2 + 7$ only function satisfying $F' = 2x$, $F(0) = 7$.

initial value problem

eg: Find $F(x)$ such that $F'(x) = \sec^2 x$, $F(0) = 9$

Antiderivative of $\sec^2 x =$ $\tan x$

$F(x) = \tan x + C$

$9 = F(0) = \tan(0) + C \rightsquigarrow C = 9$

$$\boxed{F(x) = \tan x + 9}$$

Higher Order Derivatives

eg: $F''(x) = 2x$

derivative ↑ ↓ Antiderivative

$F'(x) = \int 2x\,dx = x^2 + C$

derivative ↑ ↓ Antiderivative

$F(x) = \int x^2 + C\,dx = \frac{x^3}{3} + Cx + D$

$\underbrace{\frac{d}{dx} F'(x) = F''(x)}_{F'(x) \text{ is Antiderivative of } F''(x)}$

To find C, D we need 2 extra conditions: eg: $F(0) = 7$, $F'(0) = -3$

$7 = F(0) = 0 + 0 + D \rightsquigarrow D = 7$

$-3 = F'(0) = 0^2 + C \rightsquigarrow C = -3$

$F(x) = \frac{x^3}{3} - 3x + 7 \rightsquigarrow$ satisfies $F''(x) = 2x$, $F(0) = 7$, $F'(0) = -3$.

eg: find $F(x)$ such that $F''(x) = \sin x$, $F(0) = -11$, $F'(0) = 12$

$$\Big\downarrow \text{Antiderivative}$$

$$F'(x) = \int \sin x \, dx = -\cos x + C$$

$$\Big\downarrow \text{Antiderivative}$$

$$F(x) = \int -\cos x + C \, dx$$

$$F(x) = -\sin x + Cx + D$$

$$-11 = F(0) = -\sin(0) + C(0) + D = D \rightsquigarrow D = -11$$

$$12 = F'(0) = -\cos(0) + C = -1 + C \rightsquigarrow C = 13$$

$$\boxed{F(x) = -\sin x + 13x - 11}$$